

A NOTE ON p -ADIC VALUATIONS OF THE SCHENKER SUMS

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ABSTRACT. A prime number p is called a Schenker prime if there exists such $n \in \mathbb{N}_+$ that $p \nmid n$ and $p \mid a_n$, where $a_n = \sum_{j=0}^n \frac{n!}{j!} n^j$ is so-called Schenker sum. T. Amdeberhan, D. Callan and V. Moll formulated two conjectures concerning p -adic valuations of a_n in case when p is a Schenker prime. In particular, they asked whether for each $k \in \mathbb{N}_+$ there exists the unique positive integer $n_k < p^k$ such that $v_p(a_{m \cdot 5^k + n_k}) \geq k$ for each nonnegative integer m . We prove that for every $k \in \mathbb{N}_+$ the inequality $v_5(a_n) \geq k$ has exactly one solution modulo 5^k . This confirms the first conjecture stated by the mentioned authors. Moreover, we show that if $37 \nmid n$ then $v_{37}(a_n) \leq 1$, what means that the second conjecture stated by the mentioned authors is not true.

1. INTRODUCTION

Questions concerning the behaviour of p -adic valuations of elements of integer sequences are interesting subjects of research in number theory. The knowledge of all p -adic valuations of a given number is equivalent to its factorization. Papers [1], [2], [3], [5] and [6] present interesting results concerning behaviour of p -adic valuations in some integer sequences.

Fix a prime number p . Every nonzero rational number x can be written in the form $x = \frac{a}{b} p^t$, where $a \in \mathbb{Z}$, $b \in \mathbb{N}_+$, $\gcd(a, b) = 1$ and $p \nmid ab$. Such a representation of x is unique, thus the number t is well defined. We call t the p -adic valuation of the number x and denote it by $v_p(x)$. By convention, $v_p(0) = +\infty$. In particular, if $x \in \mathbb{Q} \setminus \{0\}$ then $|x| = \prod_{p \text{ prime}} p^{v_p(x)}$, where $v_p(x) \neq 0$ for finitely many prime numbers p . In the sequel by $s_d(n)$ we denote the sum of digits of positive integer n in base d , i.e. if $n = \sum_{i=0}^m c_i d^i$ is an expansion of n in base d , then $s_d(n) = \sum_{i=0}^m c_i$.

In a recent paper T. Amdeberhan, D. Callan and V. Moll introduced the sequence of Schenker sums, which are defined in the following way:

$$a_n = \sum_{j=0}^n \frac{n!}{j!} n^j, n \in \mathbb{N}_+.$$

The authors of [1] obtained exact expression for 2-adic valuation of Schenker sums:

$$v_2(a_n) = \begin{cases} 1, & \text{when } 2 \nmid n \\ n - s_2(n), & \text{when } 2 \mid n \end{cases}.$$

Moreover, they proved two results concerning p -adic valuation of elements of sequence $(a_n)_{n \in \mathbb{N}_+}$ when p is an odd prime number:

Proposition 1 (Proposition 3.1 in [1]). *Let p be an odd prime number and $n = pm$ for some $m \in \mathbb{N}$. Then:*

$$v_p(a_n) = \frac{n - s_p(n)}{p-1} = v_p(n!).$$

2010 *Mathematics Subject Classification.* 11B50, 11B83, 11L99.
Key words and phrases. p -adic valuation, prime, Schenker sum.

Proposition 2 (Proposition 3.2 in [1]). *Let p be an odd prime number and $n = pm + r$, where $0 < r < p$. Then $p \mid a_n$ if and only if $p \mid a_r$.*

These propositions allow us to compute p -adic valuation of a_n , when $p \mid n$ or $p \nmid a_n \bmod p$. This gives complete description of p -adic valuation of numbers a_n for some prime numbers (3, 7, 11, 17, for example). However, the question concerning p -adic valuation in case, when $p \nmid n$ and $p \mid a_n$ for some positive integer n is much more difficult. The first prime p such that $p \nmid n$ and $p \mid a_n$ for some $n \in \mathbb{N}_+$ is $p = 5$. We have $5 \mid a_{5m+2}$ for every $m \in \mathbb{N}$. Let us observe that if $n \not\equiv 0, 2 \pmod{5}$, then $5 \nmid a_n$. According to numerical experiments, the authors of the paper [1] formulated a conjecture, equivalent version of which is as follows:

Conjecture 1 (Conjecture 4.6 in [1]). *Assume that n_k is the unique natural number less than 5^k such that $5^k \mid a_{m \cdot 5^k + n_k}$, $m \in \mathbb{N}$. Then there exists the unique number $n_{k+1} \in \{n_k, 5^k + n_k, 2 \cdot 5^k + n_k, 3 \cdot 5^k + n_k, 4 \cdot 5^k + n_k\}$ such that $5^{k+1} \mid a_{m \cdot 5^{k+1} + n_{k+1}}$, $m \in \mathbb{N}$. In other words, for every $k \in \mathbb{N}$ the inequality $v_5(a_n) \geq k$ has the unique solution $n \pmod{5^k}$ with $5 \nmid n$.*

Number 5 is not the unique prime number p such that $p \nmid n$ and $p \mid a_n$ for some $n \in \mathbb{N}_+$. The prime numbers which satisfy the condition above are called *Schenker primes*.

The first question which comes to mind is: what is the cardinality of the set of Schenker primes? We will prove the following proposition using modification of the Euclid's proof of infinitude of set of prime numbers:

Proposition 3. *There are infinitely many Schenker primes.*

Proof. Assume that there are only finitely many Schenker primes and let p_1, p_2, \dots, p_s be the odd Schenker primes in ascending order. Since $a_1 = 2$, we thus obtain by proposition 2 that $p_1, p_2, p_3, \dots, p_s \nmid a_{p_1 p_2 p_3 \dots p_s + 1}$. Let us put $t := p_1 p_2 p_3 \dots p_s + 1$ and note that it is an even number. By Proposition 1 we have:

$$2t! \leq t! \sum_{j=0}^t \frac{t^j}{j!} = a_t = \prod_{p \text{ prime}, p \mid t} p^{v_p(t!)} \leq t!$$

and it leads to contradiction. \square

The main result of this paper is contained in the following theorem:

Theorem 1. *Let p be a prime number, let $n_k \in \mathbb{N}$ be such that $p \nmid n_k$, $p^k \mid a_{n_k}$ and*

$$q_{n_k, p} := a_{n_k + p} - a_{n_k} (n_k + p)^{n_k + 2} n_k^{p - n_k - 2}.$$

Then:

- *if $q_{n_k, p} \not\equiv 0 \pmod{p^2}$, then there exists unique n_{k+1} modulo p^{k+1} for which $p^{k+1} \mid a_{n_{k+1}}$ and $n_{k+1} \equiv n_k \pmod{p^k}$;*
- *if $q_{n_k, p} \equiv 0 \pmod{p^2}$ and $p^{k+1} \mid a_{n_k}$, then $p^{k+1} \mid a_{n_{k+1}}$ for any n_{k+1} satisfying $n_{k+1} \equiv n_k \pmod{p^k}$;*
- *if $q_{n_k, p} \equiv 0 \pmod{p^2}$ and $p^{k+1} \nmid a_{n_k}$, then $p^{k+1} \nmid a_{n_{k+1}}$ for any n_{k+1} satisfying $n_{k+1} \equiv n_k \pmod{p^k}$.*

Moreover, if $p \nmid n_1$, $p \mid a_{n_1}$ and $q_{n_1, p} \not\equiv 0 \pmod{p^2}$, then for any $k \in \mathbb{N}_+$ the inequality $v_p(a_n) \geq k$ has the unique solution n_k modulo p^k satisfying the congruence $n_k \equiv n_1 \pmod{p}$.

The proof of this theorem is given in section 2.

The authors of [1] stated another, more general conjecture concerning p -adic valuations of numbers a_n when p is an odd Schenker prime. The equivalent version of this conjecture is as follows:

Conjecture 2 (Conjecture 4.12. in [1]). *Let p be an odd Schenker prime. Then for every k there exists the unique solution modulo p^k of inequality $v_p(a_n) \geq k$ which is not congruent to 0 modulo p .*

Using results of Theorem 1 we will show that Conjecture 2 is not satisfied by all odd Schenker primes.

Convention. We assume that the expression $x \equiv y \pmod{p^k}$ means $v_p(x - y) \geq k$ for prime number p and the integer k . The following convention extends relation of equivalence modulo p^k to all rational numbers x, y and integers k . Moreover, we set a convention that $\prod_{i=0}^{-1} = 1$.

2. PROOF OF THE MAIN THEOREM

Theorem 1 recalls a well known fact concerning p -adic valuation of a value of polynomial with integer coefficients (see [4], page 44):

Theorem 2. *Let f be a polynomial with integer coefficients, p be a prime number and k be a positive integer. Assume that $f(n_0) \equiv 0 \pmod{p^k}$ for some integer n_0 . Then number of solutions n of the congruence $f(n) \equiv 0 \pmod{p^{k+1}}$, satisfying the condition $n \equiv n_0 \pmod{p^k}$, is equal to:*

- 1, when $f'(n_0) \not\equiv 0 \pmod{p}$;
- 0, when $f'(n_0) \equiv 0 \pmod{p}$ and $f(n_0) \not\equiv 0 \pmod{p^{k+1}}$;
- p , when $f'(n_0) \equiv 0 \pmod{p}$ and $f(n_0) \equiv 0 \pmod{p^{k+1}}$.

Similarity of these theorems is not incidental. Namely, we will show that checking p -adic valuation of values of some polynomials is sufficient for computation of the p -adic valuation of the Schenker sum. Firstly, note that for any positive integers d, n which are coprime the divisibility of a_n by d is equivalent to the divisibility of $a_{n \bmod p}$ by d :

$$\begin{aligned}
 (1) \quad a_n &= \sum_{j=0}^n \frac{n!}{j!} n^j = \sum_{j=0}^n n^{n-j} \prod_{i=0}^{j-1} (n-i) \equiv \\
 &\equiv \sum_{j=0}^{d-1} n^{n-j} \prod_{i=0}^{j-1} (n-i) = n^{n-d+2} \sum_{j=0}^{d-1} n^{d-j-2} \prod_{i=0}^{j-1} (n-i) \pmod{d},
 \end{aligned}$$

where the equivalence between the third and the fourth expression follows from the fact that the product of at least d consecutive integers contains an integer d divisible by d , hence it is equal to 0 mod d . Thus for every $d \in \mathbb{N}_+$ we define the polynomial:

$$f_d(X) := \sum_{j=0}^{d-1} X^{d-j-2} \prod_{i=0}^{j-1} (X-i).$$

With this notation the formula (1) can be rewritten in the following way:

$$(2) \quad a_n \equiv n^{n-d+2} f_d(n) \pmod{d}.$$

Let $r = n \pmod{d}$. If d, n are coprime, then the following sequence of equivalences is true:

$$\begin{aligned}
 a_n &\equiv n^{n-d+2} f_d(n) \equiv 0 \pmod{d} \iff f_d(n) \equiv 0 \pmod{d} \\
 \iff f_d(r) &\equiv 0 \pmod{d} \iff a_r \equiv r^{r-d+2} f_d(r) \equiv 0 \pmod{d}.
 \end{aligned}$$

If $d = p^k$ for some prime number p and positive integer k , then the formula (2) takes the form:

$$(3) \quad a_n \equiv n^{n-p^k+2} f_{p^k}(n) \pmod{p^k}.$$

We thus see that if $p \nmid n$, then $v_p(a_n) \geq k$ if and only if $v_p(f_{p^k}(n)) \geq k$. Moreover, for any $k_1, k_2 \in \mathbb{N}$ satisfying $k_1 \leq k_2$ the following congruence holds:

$$n^{n-p^{k_1}+2} f_{p^{k_1}}(n) \equiv n^{n-p^{k_2}+2} f_{p^{k_2}}(n) \pmod{p^{k_1}}.$$

Hence, if $p \nmid n$ and $k_1 \leq k_2$, then:

$$p^{k_1} \mid f_{p^{k_2}}(n) \iff p^{k_1} \mid f_{p^{k_1}}(n).$$

If we assume now that $k > 1$, then by Fermat's little theorem (in the form $n^{p^k} \equiv n \pmod{p}$) and the fact that product of at least p consecutive integers is divisible by p we obtain:

$$\begin{aligned} f'_{p^k}(n) &= \sum_{j=0}^{p^k-1} \left[(p^k - j - 2) n^{p^k-j-3} \prod_{i=0}^{j-1} (n-i) + n^{p^k-j-2} \sum_{h=0}^{j-1} \prod_{i=0, i \neq h}^{j-1} (n-i) \right] \equiv \\ &\equiv \sum_{j=0}^{2p-1} \left[(-j-2) n^{-j-2} \prod_{i=0}^{j-1} (n-i) + n^{-j-1} \sum_{h=0}^{j-1} \prod_{i=0, i \neq h}^{j-1} (n-i) \right] \pmod{p}. \end{aligned}$$

The formula above implies the congruence:

$$(4) \quad f'_{p^{k_1}}(n) \equiv f'_{p^{k_2}}(n) \pmod{p},$$

for $k_1, k_2 > 1$ and $p \nmid n$. Let us recall that if $f \in \mathbb{Z}[X]$, then for any $x_0 \in \mathbb{Z}$ there exists such a $g \in \mathbb{Z}[X]$ that

$$f(X) = f(x_0) + (X - x_0)f'(x_0) + (X - x_0)^2 g(X).$$

Using the formula (3) and the equality above for $f = f_{p^2}$, $x_0 = n$ and $X = n + p$, we have:

$$(5) \quad \frac{a_{n+p}}{(n+p)^{n+p-p^2+2}} - \frac{a_n}{n^{n-p^2+2}} \equiv f_{p^2}(n+p) - f_{p^2}(n) \equiv p f'_{p^2}(n) \pmod{p^2}.$$

If $p \nmid x$, then by Euler's theorem $x^{p^2-p} \equiv 1 \pmod{p^2}$ we can simplify the congruence (5) and get:

$$\frac{a_{n+p}}{(n+p)^{n+2}} - \frac{a_n}{n^{n-p+2}} \equiv p f'_{p^2}(n) \pmod{p^2}$$

and this leads to the congruence

$$\frac{1}{p} \left(\frac{a_{n+p}}{(n+p)^{n+2}} - \frac{a_n}{n^{n-p+2}} \right) \equiv f'_{p^2}(n) \pmod{p}.$$

Our consideration shows that the following conditions are equivalent:

$$\begin{aligned} & f'_{p^2}(n) \not\equiv 0 \pmod{p} \\ \iff & \frac{1}{p} \left(\frac{a_{n+p}}{(n+p)^{n+2}} - \frac{a_n}{n^{n-p+2}} \right) \not\equiv 0 \pmod{p} \\ (6) \quad \iff & \frac{a_{n+p}}{(n+p)^{n+2}} - \frac{a_n}{n^{n-p+2}} \not\equiv 0 \pmod{p^2} \\ \iff & a_{n+p} - \frac{a_n(n+p)^{n+2}}{n^{n-p+2}} \not\equiv 0 \pmod{p^2} \\ \iff & a_{n+p} - a_n(n+p)^{n+2} n^{p-n-2} \not\equiv 0 \pmod{p^2}. \end{aligned}$$

Assume now that $p^k \mid a_{n_k}$ for some $n_k \in \mathbb{N}$ not divisible by p and

$$a_{n_k+p} - a_{n_k}(n_k+p)^{n_k+2}n_k^{p-n_k-2} \not\equiv 0 \pmod{p^2}.$$

Then $p \nmid f'_{p^2}(n_k)$ and by (5) $p \nmid f'_{p^{k+1}}(n_k)$. Using now the Theorem 2 for $f = f_{p^{k+1}}$ we conclude that there exists the unique $n_{k+1} \in \mathbb{Z}$ modulo p^{k+1} satisfying conditions $p^{k+1} \mid a_{n_{k+1}}$ and $n_{k+1} \equiv n_k \pmod{p^k}$.

By simple induction on k we obtain that if $p \nmid n_1$ and $p \mid a_{n_1}$ together with the condition

$$a_{n_1+p} - a_{n_1}(n_1+p)^{n_1+2}n_1^{p-n_1-2} \not\equiv 0 \pmod{p^2},$$

then there exists the unique n_k modulo p^k such that $p^k \mid a_{n_k}$, $n_k \equiv n_1 \pmod{p}$ and

$$\frac{1}{p} \left(\frac{a_{n_k+p}}{(n_k+p)^{n_k+2}} - \frac{a_{n_k}}{n_k^{n_k-p+2}} \right) \equiv \frac{1}{p} \left(\frac{a_{n_1+p}}{(n_1+p)^{n_1+2}} - \frac{a_{n_1}}{n_1^{n_1-p+2}} \right) \pmod{p}.$$

Certainly this statement is true for $k = 1$. Now, if we assume that there exists the unique n_k modulo p^k satisfying the conditions in the statement, then there exists the unique n_{k+1} modulo p^{k+1} such that $p^{k+1} \mid a_{n_{k+1}}$, $n_{k+1} \equiv n_k \pmod{p^k}$. Using (6) we conclude that

$$\begin{aligned} \frac{1}{p} \left(\frac{a_{n_{k+1}+p}}{(n_{k+1}+p)^{n_{k+1}+2}} - \frac{a_{n_{k+1}}}{n_{k+1}^{n_{k+1}-p+2}} \right) &\equiv f'_{p^{k+1}}(n_{k+1}) \equiv f'_{p^{k+1}}(n_k) \equiv \\ &\equiv \frac{1}{p} \left(\frac{a_{n_k+p}}{(n_k+p)^{n_k+2}} - \frac{a_{n_k}}{n_k^{n_k-p+2}} \right) \equiv \frac{1}{p} \left(\frac{a_{n_1+p}}{(n_1+p)^{n_1+2}} - \frac{a_{n_1}}{n_1^{n_1-p+2}} \right) \pmod{p}. \end{aligned}$$

Summing up our discussion we see that the first case in the statement of Theorem 1 is proved. We prove the rest of the statement now.

Let $p \nmid n_k$ and $p^k \mid a_{n_k}$ and

$$a_{n_k+p} - a_{n_k}(n_k+p)^{n_k+2}n_k^{p-n_k-2} \equiv 0 \pmod{p^2}.$$

Since the last of the conditions above is equivalent to divisibility of $f'_{p^k}(n_k)$ by p , the Theorem 2 allows us to conclude that:

- if $p^{k+1} \mid a_{n_k}$, then $p^{k+1} \mid a_n$ for any $n \equiv n_k \pmod{p^k}$;
- if $p^{k+1} \nmid a_{n_k}$, then $p^{k+1} \nmid a_n$ for any $n \equiv n_k \pmod{p^k}$.

We have obtained an useful criterion for behaviour of p -adic valuation for numbers a_n . In particular, the condition:

$$a_{n_1+p} - a_{n_1}(n_1+p)^{n_1+2}n_1^{p-n_1-2} \not\equiv 0 \pmod{p^2}$$

is not only sufficient, but also necessary condition for existence of the unique solution modulo p^k of inequality $v_p(a_n) \geq k$ such that $n \equiv n_1 \pmod{p}$.

3. SOLUTION OF CONJECTURES

First of all let us note that Theorem 1 provides the formula $v_2(a_n) = 1$ for every odd positive integer n . Indeed:

$$q_{1,2} = a_3 - a_1 \cdot 3^{1+2} \cdot 1^{2-1-2} = 78 - 2 \cdot 27 = 24 \equiv 0 \pmod{4}$$

and $a_1 = 2$ and thus $2 \nmid n$, then $v_2(a_n) = 1$. This gives an alternative proof of Amdeberhan's, Callan's and Moll's result.

Theorem 1 allows us to prove that the Conjecture 1 is true by verifying the condition $a_7 - a_2 \cdot 7^{2+2} \cdot 2^{5-2-2} \not\equiv 0 \pmod{5^2}$. It is easy to check that

$$a_7 - a_2 \cdot 7^{2+2} \cdot 2^{5-2-2} = 3309110 - 10 \cdot 2401 \cdot 2 = 3261090 \equiv 15 \not\equiv 0 \pmod{25}$$

and the proof of the Conjecture 1 is finished.

Let us take the next Schenker prime $p = 13$. If $13 \nmid n$, then $13 \mid a_n$ if and only if $n \equiv 3 \pmod{13}$. Using Theorem 1 for $p = 13$ and $n_1 = 3$:

$$\begin{aligned} a_{16} - a_3 \cdot 16^{3+2} \cdot 3^{13-3-2} &= 105224992014096760832 - 78 \cdot 1048576 \cdot 6561 = \\ &= 117 - 78 \cdot 100 \cdot 139 = -1084083 \equiv 52 \pmod{169}, \end{aligned}$$

we conclude that for every positive natural k there exists the unique solution modulo 13^k of inequality $v_{13}(a_n) \geq k$ which is not divisible by p and we know that it is congruent to 3 modulo 13. This implies that if $p = 13$, then the Conjecture 1 is true.

The Conjecture 2 states that for every odd Schenker prime p there exists the unique $n_1 \in \mathbb{N}_+$ less than p such that $p \mid a_{n_1}$ and for this n_1 we have:

$$a_{n_1+p} - a_{n_1}(n_1 + p)^{n_1+2}n_1^{p-n_1-2} \not\equiv 0 \pmod{p^2}.$$

However, it is easy to see that the Conjecture 2 is not true in general. Indeed, let us put $p = 37$. If $37 \nmid n$, then $37 \mid a_n$ if and only if $n \equiv 25 \pmod{37}$. However, $37^2 \mid a_{62} - a_{25} \cdot 62^{27} \cdot 25^{10}$. Moreover, $a_{25} \equiv 851 = 23 \cdot 37 \pmod{37^2}$, thus $v_{37}(a_n) = 1$ for any $n \equiv 25 \pmod{37}$. Hence 37-adic valuation of Schenker sums is bounded by one on the set of positive natural numbers not divisible by 37. We can describe it by a simple formula:

$$v_{37}(a_n) = \begin{cases} \frac{n - s_{37}(n)}{36}, & \text{when } n \equiv 0 \pmod{37} \\ 1, & \text{when } n \equiv 25 \pmod{37} \\ 0, & \text{when } n \not\equiv 0, 25 \pmod{37} \end{cases}.$$

Our result shows that the Conjecture 2 is false for $p = 37$. Moreover, there exist prime numbers p for which number of solutions modulo p of congruence $a_n \equiv 0 \pmod{p}$, where $p \nmid n$, is greater than 1. Denote this number by $\lambda(p)$ (note that a prime number p is a Schenker prime if and only if $\lambda(p) > 0$). According to computations in Mathematica [7], we know that there are 126 Schenker primes among 200 first prime numbers. In the table below we present the solutions of the equation $\lambda(p) = k$ for $k \leq 5$.

| $\lambda(p)$ | p |
|--------------|--|
| 1 | 5, 13, 23, 31, 37, 43, 47, 53, 59, 61, 71, 79, 101, 103, 107, 109, 127, 137, 157, 163, 173, 229, 241, 251, 257, 263, 317, 337, 349, 353, 359, 397, 421, 431, 487, 499, 503, 521, 547, 571, 577, 587, 617, 619, 641, 653, 661, 727, 733, 757, 797, 811, 821, 829, 881, 883, 937, 947, 967, 977, 991, 1013, 1031, 1039, 1091, 1097, 1123, 1163, 1181, 1213 |
| 2 | 41, 149, 181, 191, 199, 211, 271, 283, 293, 311, 367, 383, 401, 409, 419, 439, 461, 523, 541, 563, 569, 607, 613, 647, 673, 691, 709, 761, 787, 827, 929, 941, 983, 997, 1021, 1051, 1061, 1087, 1117, 1151, 1153, 1223 |
| 3 | 179, 197, 223, 277, 509, 601, 683, 743, 887, 1201 |
| 4 | — |
| 5 | 593 |

Table 1

4. QUESTIONS

Although Conjecture 2 is not true, we do not know if 2 and 37 are the only primes p such that $p \mid a_n$ and $q_{n,p} = a_{n+p} - a_n(n+p)^{n+2}n^{p-n-2} \equiv 0 \pmod{p^2}$ for some $n \in \mathbb{N}_+$ which is not divisible by p . According to a numerical computations, they are unique among all primes less than 30000. The results above suggest to formulate the following questions:

Question 1. *Is there any Schenker prime greater than 37 for which there exists $n \in \mathbb{N}_+$ such that $p \nmid n$, $p \mid a_n$ and $q_{n,p} \equiv 0 \pmod{p^2}$?*

Question 2. *Are there infinitely many Schenker primes p for which there exists $n \in \mathbb{N}_+$ such that $p \nmid n$, $p \mid a_n$ and $q_{n,p} \equiv 0 \pmod{p^2}$?*

In the light of the results presented in the table some natural questions arise:

Question 3. *Are there infinitely many Schenker primes p for which $\lambda(p) > 1$?*

Question 4. *Let m be a positive integer. Is there any Schenker prime p such that $\lambda(p) \geq m$?*

In section 1 of this paper we presented a short proof of the infinitude of set of Schenker primes. In view of this fact it is natural to ask:

Question 5. *Are there infinitely many primes which are not Schenker primes?*

ACKNOWLEDGEMENTS

I wish to thank my MSc thesis advisor Maciej Ulas for many valuable remarks concerning presentation of results. I would like also to thank Maciej Gawron for help with computations and Tomasz Pełka for help with edition of the paper.

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